12.6

*Or* and *And* Problems
Or Problems

- \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)
- **Example:** Each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is written on a separate piece of paper. The 10 pieces of paper are then placed in a bowl and one is randomly selected. Find the probability that the piece of paper selected contains an even number or a number greater than 5.
Solution

- \[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
- \[ P\left( \text{even or greater than 5} \right) = P(\text{even}) + P(\text{greater than 5}) - P(\text{even and greater than 5}) \]
- \[ = \frac{5}{10} + \frac{5}{10} - \frac{3}{10} \]
- \[ = \frac{7}{10} \]

Thus, the probability of selecting an even number or a number greater than 5 is 7/10.
Example

Each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is written on a separate piece of paper. The 10 pieces of paper are then placed in a bowl and one is randomly selected. Find the probability that the piece of paper selected contains a number less than 3 or a number greater than 7.
Solution

\[ P(\text{less than 3}) = \frac{2}{10} \]
\[ P(\text{greater than 7}) = \frac{3}{10} \]

There are no numbers that are both less than 3 and greater than 7. Therefore,

\[ \frac{2}{10} + \frac{3}{10} - 0 = \frac{5}{10} = \frac{1}{2} \]
Mutually Exclusive

Two events $A$ and $B$ are mutually exclusive if it is impossible for both events to occur simultaneously.
Example

One card is selected from a standard deck of playing cards. Determine the probability of the following events.

- a) selecting a 3 or a jack
- b) selecting a jack or a heart
- c) selecting a picture card or a red card
- d) selecting a red card or a black card
Solutions

- a) 3 or a jack

\[ P(3) + P(\text{jack}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \]

- b) jack or a heart

\[ P(\text{jack}) + P(\text{heart}) - P(\text{jack and heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} \]
Solutions continued

- c) picture card or red card

\[ P(\text{picture}) + P(\text{red}) - P\left(\frac{\text{picture} \& \text{red card}}{\text{red card}}\right) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13} \]

- d) red card or black card

\[ P(\text{red}) + P(\text{black}) = \frac{26}{52} + \frac{26}{52} = \frac{52}{52} = 1 \]
Independent Events

- Event A and Event B are **independent events** if the occurrence of either event in no way affects the probability of the occurrence of the other event.

- Experiments done with replacement will result in independent events, and those done without replacement will result in dependent events.
And Problems

- If A and B are independent,
  \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

- Example: Two cards are to be selected with replacement from a deck of cards. Find the probability that two red cards will be selected.

  \[
  P(A) \cdot P(B) = P(\text{red}) \cdot P(\text{red}) \\
  = \frac{26}{52} \cdot \frac{26}{52} \\
  = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}
  \]
Example

Two cards are to be selected without replacement from a deck of cards. Find the probability that two red cards will be selected.

\[ P(A) \cdot P(B) = P(\text{red}) \cdot P(\text{red}) \]

\[ = \frac{26}{52} \cdot \frac{25}{52} \]

\[ = \frac{1}{2} \cdot \frac{25}{52} = \frac{25}{104} \]
Example

- A package of 30 tulip bulbs contains 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers. Three bulbs are randomly selected and planted. Find the probability of each of the following.
  - All three bulbs will produce pink flowers.
  - The first bulb selected will produce a red flower, the second will produce a yellow flower and the third will produce a red flower.
  - None of the bulbs will produce a yellow flower.
  - At least one will produce yellow flowers.
Solution

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

All three bulbs will produce pink flowers.

\[
P(3 \text{ pink}) = P(\text{pink 1}) \cdot P(\text{pink 2}) \cdot P(\text{pink 3})
\]

\[
= \frac{6}{30} \cdot \frac{5}{29} \cdot \frac{4}{28}
\]

\[
= \frac{1}{203}
\]
Solution

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

The first bulb selected will produce a red flower, the second will produce a yellow flower and the third will produce a red flower.

\[
P(\text{red, yellow, red}) = P(\text{red}) \cdot P(\text{yellow}) \cdot P(\text{red})
\]
\[
= \frac{14}{30} \cdot \frac{10}{29} \cdot \frac{13}{28}
\]
\[
= \frac{13}{174}
\]
Solution

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

None of the bulbs will produce a yellow flower.

\[
P(\text{none yellow}) = P(\text{first not yellow}) \cdot P(\text{second not yellow}) \cdot P(\text{third not yellow})
\]

\[
= \frac{20}{30} \cdot \frac{19}{29} \cdot \frac{18}{28}
\]

\[
= \frac{57}{203}
\]
Solution

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

At least one will produce yellow flowers.

\[ P(\text{at least one yellow}) = 1 - P(\text{no yellow}) \]
\[ = 1 - \frac{57}{203} \]
\[ = \frac{146}{203} \]
12.7

Conditional Probability
Conditional Probability

- In general, the probability of event $E_2$ occurring, given that an event $E_1$ has happened (or will happen; the relationship does not matter) is called **conditional probability** and is written $P(E_2|E_1)$. 
Example

Given a family of two children, and assuming that boys and girls are equally likely, find the probability that the family has

- a) two girls.
- b) two girls if you know that at least one of the children is a girl.
- c) two girls given that the older child is a girl.
Solutions

a) two girls

There are four possible outcomes BB, BG, GB, and GG.

\[ P(2 \text{ girls}) = \frac{1}{4} \]

b) two girls if you know that at least one of the children is a girl

\[ P(\text{both girls}|\text{at least one is a girl}) = \frac{1}{3} \]
Solutions continued

- two girls given that the older child is a girl

\[ P(\text{both girls}|\text{older child is a girl}) = \frac{1}{2} \]
Conditional Probability

For any two events, $E_1$, and $E_2$,

$$P(E_2 \mid E_1) = \frac{n(E_1 \text{ and } E_2)}{n(E_1)}$$
Example

Use the results of the taste test given at a local mall. If one person from the sample is selected at random, find the probability the person selected

<table>
<thead>
<tr>
<th></th>
<th>Prefers Peppermint</th>
<th>Prefers Wintergreen</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>70</td>
<td>55</td>
<td>125</td>
</tr>
<tr>
<td>Women</td>
<td>60</td>
<td>72</td>
<td>132</td>
</tr>
<tr>
<td>Total</td>
<td>130</td>
<td>127</td>
<td>257</td>
</tr>
</tbody>
</table>
Example continued

- a) prefers peppermint
  \[ P(\text{peppermint}) = \frac{130}{257} \]

- b) is a woman
  \[ P(\text{woman}) = \frac{132}{257} \]
Example continued

- c) prefers peppermint, given that a woman is selected
  
  \[ P(\text{peppermint}|\text{woman}) = \frac{60}{132} = \frac{15}{33} \]

- d) is a man, given that the person prefers wintergreen
  
  \[ P(\text{man}|\text{wintergreen}) = \frac{55}{127} \]
Next Steps

- Read all examples in the text for the corresponding sections.
- Homework problems from text:
  - 12.6, p. 705: 27-43, odds; 63-66, all
  - 12.7, p. 710: 5-23, odds; 47-52, all; 65-70, all
- Do Online Homework
- Do online quiz after finishing homework for both sections