KINEMATICS IN TWO DIMENSION

• Projectile Motion
  – Projectile motion is the motion of an object fired at an angle $\theta$ with the horizontal.
  – This motion can be discussed by analyzing the horizontal components of the object's motion independently of the vertical component of motion.

\[ v_{0x} = v_0 \cos \theta \]
\[ v_{0y} = v_0 \sin \theta \]

KINEMATICS IN TWO DIMENSION...

• Projectile Motion...
  – If air resistance is negligible, then the horizontal component of motion does not change $\Rightarrow$ constant velocity motion
  – The following equation describes the horizontal motion of a projectile.

\[ x = v_x t \]
KINEMATICS IN TWO DIMENSION...

- Projectile Motion...
  - The vertical component of motion is affected by gravity ⇒ uniform constant acceleration motion
  - The motion is then described by the equations for an object in free fall.
  - The following equations are used to describe the vertical motion of a projectile.

\[
\begin{align*}
  y &= v_{0y} t + \frac{1}{2} (-g) t^2 \\
  v_y &= v_{0y} + (-g) t \quad \Rightarrow \\
  v_y^2 &= v_{0y}^2 + 2(-g)y
\end{align*}
\]

- Problem 1
  - In a particular laboratory experiment, a spring gun placed on a table fires a steel ball horizontally outward. A student determines that the ball starts 1.0 m above the floor and travels 2.7 m horizontally before it strikes the floor. Determine the
  - a) time that the ball is in the air and
  - b) initial velocity of the ball.

\[
\begin{align*}
x &= 2.7 \text{ m} \\
y &= -1.0 \text{ m} \\
 v_{0x} &= v_0 \cos \theta = v_0 \\
 v_{0y} &= v_0 \sin \theta = 0
\end{align*}
\]

\[
\begin{align*}
a) \quad y &= v_{0y} t + \frac{1}{2} (-g) t^2 \\
&\Rightarrow t = 0.45 \text{ s}
\end{align*}
\]

\[
\begin{align*}
b) \quad x &= v_{0x} t \\
&\Rightarrow v_{0x} = v_0 = 6.0 \text{ m/s}
\end{align*}
\]
**KINEMATICS IN TWO DIMENSION...**

**Problem 2**

- A spring gun placed on a table fires a steel ball at a $45^\circ$ angle above the horizontal. The ball leaves the muzzle of the gun 1.1 m above the floor and travels 4.6 m horizontally. Determine the
- a) total time that the ball is in the air.

$$
\theta = 45^\circ \quad v_{0x} = v_0 \cos \theta = \frac{\sqrt{2}}{2} v_0 \\
y = -1.1 \text{ m} \quad v_{0y} = v_0 \sin \theta = \frac{\sqrt{2}}{2} v_0 \\
y = v_{0y} t + \frac{1}{2} (-g) t^2 \quad \Rightarrow \begin{cases} -1.1 = \frac{\sqrt{2}}{2} v_0 t - 4.9 t^2 \\ 4.6 = \frac{\sqrt{2}}{2} v_0 t \end{cases} \\
x = v_{0x} t \quad \Rightarrow t = 1.08 \text{ s}
$$

**Problem 3**

- A projectile is fired with an initial speed of 113 m/s at an angle of $60.0^\circ$ above the horizontal from the top of a cliff 49.0 m high. Determine the
- a) time to reach maximum height,
- b) maximum height above the base of the cliff reached by the projectile

$$
v_0 = 113 \text{ m/s} \quad v_{0x} = v_0 \cos \theta = 56.5 \text{ m/s} \\
g = 9.8 \text{ m/s}^2 \quad v_{0y} = v_0 \sin \theta = 97.9 \text{ m/s} \\
a) \quad \Rightarrow v_y = v_{0y} + (-g) t \quad \Rightarrow t = 10.0 \text{ s} \\
b) \quad \Rightarrow y = v_{0y} t + \frac{1}{2} (-g) t^2 \quad \Rightarrow y = 489 \text{ m} \\
b) \quad \Rightarrow h = 489 \text{ m} + 49 \text{ m} \quad \Rightarrow h = 538 \text{ m}
$$
**KINEMATICS IN TWO DIMENSION...**

- **Problem 4**
  - A projectile is fired with an initial speed of 113 m/s at an angle of $60.0^\circ$ above the horizontal from the top of a cliff 49.0 m high. Determine the
  - a) total time it stays in the air, and
  - b) horizontal range of the projectile.

  \[ v_0 = 113 \text{ m/s} \]
  \[ \theta = 60.0^\circ \]
  \[ y = -49 \text{ m} \]

  \[ v_0_x = v_0 \cos \theta = 56.5 \text{ m/s} \]
  \[ v_0_y = v_0 \sin \theta = 97.9 \text{ m/s} \]

  \( a) \rightarrow y = v_0_y t + \frac{1}{2} (-g) t^2 \quad \Rightarrow t = 20.5 \text{ s} \)

  \( b) \rightarrow x = v_x t \quad \Rightarrow x = 1156 \text{ m} \)

- **Problem 5**
  - A stone is thrown horizontally outward from the top of a bridge. The stone is released 19.6 m above the street below. The initial velocity of the stone is 5.0 m/s. Determine the
  - a) total time that the stone is in the air and
  - b) magnitude and direction of the velocity of the projectile “just” before it strikes the street.

  \[ v_0 = 5.0 \text{ m/s} \]
  \[ y = -90.6 \text{ m} \]
  \[ g = 9.8 \text{ m/s}^2 \]

  \[ v_0_x = v_0 \cos \theta = 5.0 \text{ m/s} \]
  \[ v_0_y = v_0 \sin \theta = 0 \]

  \( a) \rightarrow y = v_0_y t + \frac{1}{2} (-g) t^2 \quad \Rightarrow t = 2.0 \text{ s} \)

  \( b) \rightarrow v_y = v_0_y + (-g) t \quad \Rightarrow v_y = -19.6 \text{ m/s} \)

  \( b) \rightarrow v = \sqrt{v_x^2 + v_y^2} \quad \Rightarrow v = 20.0 \text{ m/s} \)

  \( b) \rightarrow \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) \quad \Rightarrow \theta = 76^\circ \)

The direction is $76^\circ$ south of east
• Relative Velocity
  – Relative velocity refers to the velocity of an object with respect to a particular frame of reference.
  – The reference frame is usually specified by using Cartesian coordinates, i.e., x and y axis, relative to which the position and/or motion of an object can be determined.

\[
\vec{V}_{BS} = \vec{V}_{BW} + \vec{V}_{WS}
\]

**KINEMATICS IN TWO DIMENSION...**

• Relative Velocity...
  – The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference and the relative velocity of the two reference frames are known.
General Physics

**Problem 6**
- The current in the river is 1.0 m/s. A woman swims 300 m down stream and then back to her starting point without stopping. If she can swim 2.0 m/s is still water, determine the time required for the round trip.

**KINEMATICS IN TWO DIMENSION...**

- **downstream**
  \[
  V_{PS} = V_{PW} + V_{WS} \quad \Rightarrow \quad V_{PS} = 2.0 + 1.0 \quad \Rightarrow \quad V_{PS} = 3.0 \text{ m/s}
  \]

  \[
  x = vt \quad \Rightarrow \quad t = 100 \text{s}
  \]

- **upstream**
  \[
  V_{PS} = V_{PW} + V_{WS} \quad \Rightarrow \quad V_{PS} = 2.0 + (-1.0) \quad \Rightarrow \quad V_{PS} = 1.0 \text{ m/s}
  \]

  \[
  x = vt \quad \Rightarrow \quad t = 300 \text{s}
  \]

  \[
  \Rightarrow \text{time} = 100 + 300 = 400 \text{s}
  \]

**Problem 7**
- The woman in the previous problem swims perpendicular across the river to the opposite bank. If the river is 300 m wide, determine
  - a) the woman velocity relative to the shore,
  - b) the distance swept downstream and
  - c) the time required to swim across the river.

**KINEMATICS IN TWO DIMENSION...**

- **downstream**
  \[
  V_{PS} = V_{PW} + V_{WS} \quad \Rightarrow \quad V_{PS} = \sqrt{(V_{PW})^2 + (V_{WS})^2} \quad \Rightarrow \quad V_{PS} = 2.2 \text{ m/s}
  \]

  \[
  \Rightarrow \theta = \tan^{-1}\left(\frac{V_{WS}}{V_{PW}}\right) \quad \Rightarrow \theta = 27^\circ
  \]

  \[
  \tan 27^\circ = \frac{y}{300} \quad \Rightarrow \quad y = 152.8
  \]

  \[
  x = v_x t \quad \Rightarrow \quad 300 = 2t \quad \Rightarrow \quad t = 150 \text{s}
  \]