(1) Show that if $a|b$ and $b|a$, where $a$ and $b$ are integers, then $a = b$ or $a = -b$.

(2) Show that if $a$, $b$, $c$, $d$ are integers such that $a|c$ and $b|d$, then $ab|cd$.

(3) Show that if $a$, $b$, and $c$ are integers such that $ac|bc$, then $a|b$.

(4) Show that if $a$ is an even integer and $b$ is an odd integer, then $ab$ is even.

(5) Show that the product of two odd integers is odd.

(6) Show that if the product of two integers $a$ and $b$ is odd, then both $a$ and $b$ must be odd. (Hint: use proof by contradiction.)

(7) Show that if the product of two integers $a$ and $b$ is even, then at least one of $a$ or $b$ must be even. (Hint: use proof by contradiction.)

(8) Which of the following are true? Justify your answer.
   (a) $\forall x \in \mathbb{R}, x^2 > 0$.
   (b) $\exists x \in \mathbb{R}$, such that $x^2 > 0$.
   (c) $\exists x \in \mathbb{Z}$ such that $4x^2 - 1 = 0$.
   (d) $\forall x \in \mathbb{R}, \left| \frac{1}{x} \right| < 10$.
   (e) $\exists x \in \mathbb{R}, \left| \frac{1}{x} \right| < 10$.

(9) Given a polynomial $p$, let $A$ be the sum of the coefficients of the even powers, and let $B$ be the sum of the coefficients of the odd powers. Prove that $A^2 - B^2 = p(1)p(-1)$.

(10) In simpler language, describe the meaning of the following two statements. Do they mean the same thing? Why or why not?
   (a) There is a number $M$ such that, for every $x$ in the set $S$, $|x| \leq M$.
   (b) For every $x$ in the set $S$, there is a number $M$ such that $|x| \leq M$. 